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KALUZA-KLEIN TYPE COSMOLOGICAL MODEL OF THE UNIVERSE WITH VACUUM ENERGY DENSITY

Sweeti B. Rokde[*1], M. R. Pund[2] and D R Waje[3]
[*1,2]Priyadarshini College of Engineering, Nagpur
[3]K D K College of Engineering, Umrer.

ABSTRACT

In this paper by considering, the Decay law Λ of the form $\Lambda = 3\alpha \frac{\dot{R}^2}{R^2} + \beta \frac{\dot{R}}{R}, \;\; \alpha, \beta = constt.$ We generalized the result of Arbab (2001) for higher dimensional space time and observed that the cosmological constant decreases as t^{-2} and rate of particle creation is smaller than the steady state value. We have found an inflationary solution of the de-Sitter type with $\beta = 6 - 3\alpha$. The present model could resolve many of standard model problems with observations and thus could become a viable candidate as an alternative model.

Keywords *cosmology, space time, Decay law.*

I. INTRODUCTION

This paper is the straight forward generalizations of the work obtained earlier by Arbab (2001) in the Kaluza-Klein theory of gravitation.In this chapter, we suggest a variational law for the cosmological constant of the form $\Lambda = 3\alpha \frac{\dot{R}^2}{R^2} + \beta \frac{\dot{R}}{R}$, where α and β are dimensionless constants. It is observed that the cosmological constant Λ is found to be decreased as t-2 and the rate of particle observation is smaller than the steady state values. It is also observed that the universe must be accelerated if Λ > 0. The model obtained here is free from a lot of cosmological problems and could fit well with the present observational data.

1.2. Metric and field equations

Consider a Robertson Walker metric

$$\mathrm{ds}^2 = \mathrm{dt}^2 - \mathrm{R}^2(\mathrm{t})(\mathrm{dx}^2 + \mathrm{dy}^2 + \mathrm{dz}^2) - \mathrm{A}^2(\mathrm{t})\mathrm{dm}^2 \quad (1.1)$$

The Einstein's field equations for the metric (1.1) with variable cosmological and gravitational 'constants' and a perfect fluid yield,

$$3(n+1)\frac{\dot{R}^2}{R^2} = 8\pi G\rho + \Lambda \quad (1.2)$$

$$3(n+1)\frac{\ddot{R}}{R} = -8\pi G[\rho + p(n+1)] + \Lambda_n \quad (1.3)$$

Where ρ is the fluid energy density and p it's pressure.

The equation of state is usually given by,

$$p = \omega\rho \quad (1.4)$$

where, ω is the equation of state parameter.

The energy conservation equation is $T^i_{j;i} = 0$, leads to,

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$$8\pi\left[G_{;i}T_j^i + GT_{j;i}^i\right] + \Lambda_{;i}\delta_j^i = 0$$

$$8\pi\left[\frac{\dot{G}}{G}\rho + \dot{\rho} + \left(3\frac{\dot{R}}{R} + \frac{\ddot{A}}{A}\right)(p+\rho)\right] + \dot{\Lambda} = 0$$

$$\dot{\rho} + \frac{\dot{G}}{G}\rho + \left(3\frac{\dot{R}}{R} + \frac{\ddot{A}}{A}\right)(p+\rho) + \frac{\dot{\Lambda}}{8\pi G} = 0$$

$$\dot{\rho} + \frac{\dot{G}}{G}\rho + (n+3)\frac{\dot{R}}{R}(P+\rho) = -\frac{\dot{\Lambda}}{8\pi G} \qquad (A = R^n)$$

$$(n+3)\dot{R}(p+\rho) = -\left(\frac{\dot{G}}{G}\rho + \dot{\rho} + \frac{\dot{\Lambda}}{8\pi G}\right)R \tag{1.5}$$

1.3 Particle Creation:

1.3.1 Matter-*dominated universe (MDU)*

For a pressure less MDU(p = 0) and for a constant, Gravitational Constant (G), equation (1.5) reads,

$$(n+3)\dot{R}\rho + \dot{\rho}R = -\frac{\dot{\Lambda}}{8\pi G}R$$

Multiply both side of above equation by R^{n+2} to get

$$(n+3)\dot{R}R^{n+2}\rho + \dot{\rho}R^{n+3} = -\frac{\dot{\Lambda}}{8\pi G}R^{n+3}$$

$$\frac{d}{dt}\left[\rho R^{n+3}\right] = -\frac{R^{n+3}}{8\pi G}\cdot\frac{d\Lambda}{dt} \tag{1.6}$$

Now, consider decay law of the form

$$\Lambda = 3\alpha\frac{\dot{R}^2}{R^2} + \beta\frac{\ddot{R}}{R} \tag{1.7}$$

Where $\propto, \beta$ are dimensionless constants. We suggest for a linear relationship between Λ and the Ricci scalar in the Einstein field equation that Λ has a general variation which resembles this scalar. For Flat space one gets the above variation.

For a flat universe, equations (1.3) and (1.7) yields,

$$[3 - \beta(1+\omega)]\ddot{R}R - 3[\alpha(1+\omega) - 1 - \omega(n+1)]\dot{R}^2 \tag{1.8}$$

which can be integrated to give

$$R(t) = \left[\frac{16 - (3\alpha+\beta)(1+\omega) + 3\omega(n+1)}{3 - \beta(1+\omega)}K_1 t\right] \tag{1.9}$$

Where k_1 is an integrating constant.

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To find $\Lambda(t)$ we have, $R(t) = (Bk_1 t)^{1/\beta}$

Where $B = \frac{[6-(3\alpha+\beta)(1+\omega)+3\omega(n+1)]}{3-\beta(1+\omega)}$

Then, $\frac{\ddot{R}}{R} = \frac{1}{Bt^2}\left(\frac{1}{B}-1\right)$ and $\frac{\dot{R}}{R} = \frac{1}{Bt}$

$$\Lambda(t) = 3\alpha\frac{1}{B^2t^2} + \beta\frac{1}{Bt^2}\left(\frac{1}{B}-1\right)$$

So that, $$\Lambda(t) = \frac{3[3-\beta(1+\omega)][3\alpha-\beta\{1+\omega(n+1)\}]}{[6-(3\propto+\beta)(1+\omega)+3\omega(n+1)]^2\,t^2} \quad (1.10)$$

Now we find $\rho(t)$:-

From Equations (1.2), (1.7) and (1.9), the energy density takes the form

$$\rho(t) = \frac{3[3-\beta(1+\omega)][3(n+1)-3\alpha-n\beta]}{8\pi G[6-(3\propto+\beta)(1+\omega)+3\omega(n+1)]^2\,t^2} \quad (1.11)$$

The vacuum energy density (ρ_V) is given by

$$\rho_v(t) = \frac{\Lambda}{8\pi G} = \frac{3}{8\pi G}\frac{[3-\beta(1+\omega)][3\alpha-\beta\{1+\omega(n+1)\}]}{[6-(3\propto+\beta)(1+\omega)+3\omega(n+1)]^2\,t^2} \quad (1.12)$$

The deceleration parameter (q) is defined as,

$$q = -\frac{\ddot{R}\,R}{\dot{R}^2} = \Lambda/8\pi G = \frac{3-3\alpha(1+\omega)+3\omega(n+1)}{3-\beta(1+\omega)} \quad (1.13)$$

Hence for Matter-Dominated Universe (MDU) i.e. $p = 0$ (ω=0), we obtain

$$R(t) = \left[\frac{6-(3\alpha+\beta)}{3-\beta}k_1 t\right]^{\frac{3-\beta}{6-(3\alpha+\beta)}} \quad (1.14)$$

$$\Lambda(t) = \frac{3(3-\beta)(3\alpha-\beta)}{[6-(3\alpha+\beta)]^2 t^2} \quad (1.15)$$

$$\rho(t) = \frac{3[3-\beta][3(n+1)-3\alpha-n\beta]}{8\pi G[6-(3\propto+\beta)]^2\,t^2} \quad (1.16)$$

$$\rho_v(t) = \frac{3}{8\pi G}\frac{[3-\beta][3\alpha-\beta]}{[6-(3\propto+\beta)]^2\,t^2} \quad (1.17)$$

and $$q = \frac{3-3\propto}{3-\beta}, \quad \beta \neq 3 \quad (1.18)$$

The density parameter of the universe (Ω^m) is given by

$$\Omega^m = \frac{\rho}{\rho_c} = \rho\frac{8\pi G}{3(n+1)H^2}$$

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$$\Omega^m = \frac{[3(n+1)-3\alpha-n\beta]}{[(n+1)(3-\beta(1+\omega)]} \qquad (1.19)$$

Hence for MDU (ω = 0)

$$\Omega^m = \frac{[3(n+1)-3\alpha-n\beta]}{[(n+1)(3-\beta)]} \qquad ,\beta \neq 3 \qquad (1.20)$$

where $\rho_c = \frac{3(n+1)H^2}{8\pi G}$ is the critical energy density of the universe and $H = \frac{\dot{R}}{R}$ is the Hubble Constant.

The density parameter due to vacuum contribution is defined as $\Omega^\Lambda = \frac{\Lambda}{3(n+1)H^2}$

Using Equations (1.9) and (1.10), this yields $\Omega^\Lambda = \frac{[3\alpha-\beta\{1+\omega(n+1)\}]}{(n+1)[3-\beta(1+\omega)]}$ (1.21)

Hence for MDU (ω = 0),we obtain,

$$\Omega^\Lambda = \frac{3\alpha-\beta}{(n+1)(3-\beta)}, \qquad \beta \neq 3 \qquad (1.22)$$

We shall define $\Omega_{total} = \Omega^m + \Omega^\Lambda$ *and* $\rho_{total} = \rho + \rho_v$ (1.23)

Hence equations (1.20),(1.22) and (1.23) gives $\Omega_{total} = 1$.

This situation is favoured by the inflationary scenario.

Also we have

$$t_p = \frac{3-\beta}{6-3\alpha-\beta} H_p^{-1}, \qquad (1.24)$$

$$\Omega_p^m = \frac{[3(n+1)-3\alpha-n\beta]}{[(n+1)(3-\beta)]} \qquad (1.25)$$

$$\Lambda_p = \frac{3(3\alpha-\beta)}{(3-\beta)} H_p^2 \quad , \quad \beta \neq 3 \qquad (1.26)$$

(Here the subscript 'p' denotes the present value of the quantity).

For ages larger than the Standard Model, one requires $\beta < 3\alpha$ and for $t_p > 0$, $\beta < 3$ and $\alpha > 1$. This constraint indicates that Λ is positive. The precise value of α *and* β has to be determined from observational data.

We now calculate the rate of particle creation (annihilation) N, which is defined as,

$$N = \frac{1}{R_p^{(n+3)}} \frac{d\{\rho R^{n+3}\}}{dt}\bigg|_p \qquad (1.27)$$

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Using equations (1.6), (1.7), (1.10), (1.24) and (1.26), one obtains

$$N_p = \frac{2(3\alpha-\beta)}{3-\beta}\rho_p\ H_p\ ,\qquad \beta \neq 3 \tag{1.28}$$

We remark that this rate is less than that of the steady state model$\left(= 3\rho_0\ H_p\ \right)$.

If $\beta = 3\alpha$ then $\Lambda = 0,\ N_p = 0,\ t_p = \frac{1}{2}H_p^{-1}$ and $\Omega_p^m = 1$. This case is equivalent to standard model result.

1.3.2 Radiation-dominated universe (RDU)

This is characterized by the equation of the state $p = \omega\rho\ (\omega = \frac{1}{3})$

In this Case, Equations (1.2), (1.3) and (1.7) yield

$$3[(n+4) - 4\alpha]\frac{\dot{R}^2}{R^2} + (9-4\beta)\frac{\ddot{R}}{R} = 0 \tag{1.29}$$

This can be solved to give

$$R(t) = \left[\frac{3(n+7)-4(3\alpha+\beta)}{9-4\beta}Dt\right]^{\frac{9-4\beta}{3(n+7)-4(3\alpha+\beta)}} \quad ,\ D=\text{constant} \tag{1.30}$$

Also equations (1.2),(1.5),(1.7) and (1.30) gives

$$\Lambda(t) = \frac{3(9-4\beta)[9\alpha-\beta(n+4)]}{[3(n+7)-4(3\alpha+\beta)]^2t^2} \tag{1.31}$$

$$\rho(t) = \frac{9}{8\pi G}\frac{(9-4\beta)[3(n+1)-3\alpha-n\beta]}{[3(n+7)-4(3\alpha+\beta)]^2t^2} \tag{1.32}$$

1.4 No particle creation

We now consider a model in which both G and Λ vary with time in such a way that the usual energy conservation law holds.

1.4.1 *Matter-dominated Universe (MDU)*:

Equation (1.5) can be written as

$$(n+3)\frac{\dot{R}}{R}(p+\rho) + \dot{\rho} = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G}\right)$$

Now by conservation equation we have, $\quad T^i_{j;i} = 0$

i.e. $$(n+3)\frac{\dot{R}}{R}(p+\rho) + \dot{\rho} = 0$$

$$(n+3)(p+\rho)H + \dot{\rho} = 0$$

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$$(n+3)\rho H + \dot{\rho} = 0 \qquad \text{(for } MDU,\ p = 0\text{)}$$

Thus we have,

$$\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{8\pi G} = 0$$

$$8\pi\dot{G}\,\rho + \dot{\Lambda} = 0$$

Thus Equation (1.5) can be split to give,

$$(n+3)\rho H + \dot{\rho} = 0 \qquad (1.33)$$

$$8\pi\dot{G}\,\rho + \dot{\Lambda} = 0 \qquad (1.34)$$

Using equations (1.9),(1.10),(1.33) and (1.34) yield

$$R(t) = \left[\frac{6-(3\alpha+\beta)k_1 t}{3-\beta}\right]^{\frac{3-\beta}{6-(3\alpha+\beta)}} \qquad (1.35)$$

$$\Lambda(t) = \left[\frac{3(3-\beta)(3\alpha-\beta)}{\left(6-(3\alpha+\beta)\right)^2 t^2}\right] \qquad (1.36)$$

$$\rho(t) = F t^{\frac{-(n+3)(3-\beta)}{[6-(3\alpha+\beta)]^2}} \quad , \quad \text{F=constant} \qquad (1.37)$$

$$G(t) = \frac{(9-3\beta)t^{\frac{2(3\alpha-\beta)}{(6-3\alpha-\beta)}}}{8\pi F(6-3\alpha-\beta)} \qquad (1.38)$$

Equation (1.34) represents a coupling between vacuum and gravity and that the vacuum decays to strengthen the gravitation interaction that will include an acceleration of the expansion of the universe. Hence, as long as gravity is increasing the expansion of the universe will continue. The variation of G could have been overwhelming in the early universe. This big gravitational force might have been the course for stopping the rapid expansion during inflationary period and latter assist in making the universe matter dominated. This because the increasing gravity forces smaller particles to form bigger ones.

For $\beta = 0,\ \alpha = 0,\ G = constant$ and $\rho = Dt^{-2}$ and $R = [2k_1 t]^{1/2}$ which is familiar FRW result. Moreover the case $\beta = 3\alpha$ is equivalent to the Standard Model result. Clearly for $\beta < 3,\ \alpha > 1$ the gravitational constant increase with time. In an earlier work (Arbab, 1997), we have considered the effect of bulk viscosity in variable G and Λ models. We have shown that many of non-viscous models are equivalent to viscous models.

1.4.2 Radiation-*dominated Universe (RDU)*:

For ω = 1/3, Equation (1.5) gives

$$\frac{4}{3}H\rho(n+3) + \dot{\rho} = 0 \qquad (1.39)$$

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And $8\pi\dot{G}\rho + \dot{\Lambda} = 0$ (1.40)

Thus, Equations (1.39) and (1.40) yield

$$\rho(t) = Mt^{\frac{-4/3(n+3)(9-4\beta)}{18-4(3\alpha+\beta+=3(n+1)}} \quad , \ \mathrm{M = constant} \tag{1.41}$$

And $$G(t) = \frac{3(9-4\beta)[9\alpha-\beta(n+4)]}{4\pi M[18-4(3\alpha+\beta)+3(n+1)]^2}\, t^{\frac{2[9(n-1)-4\beta(2n+3)+36\alpha]}{[18-4(3\alpha+\beta)+3(n+1)]}} \tag{1.42}$$

Abdel Rahman (1990) has recently considered a closed universe model with a critical energy density where both G and Λ are variables. He found that $R \propto t, G \propto t^2, \rho \propto t^{-4}$ in the radiation era. His solution corresponds to and a free β. Thus both model, albeit different; evolve in a similar way in the early universe.

1.4.3 Static *Solutions*

A static solution can be obtained for both matter and radiation dominated universes with β = 3 and β = 9/4 respectively. Thus R = constant, Λ = 0, $\rho_{\text{total}} = 0$, N = 0

It has been claimed by Kalligas et al. (1992) that they have obtained a static universe with variable G and Λ. In fact, their solution is nothing but the above solution, since with R = constant Equations (1.2) and (1.3) give Λ = 0 so that G = constant. Thus their claim of static solution with variable G and Λ can not be true with $p \neq -\rho$.

1.5 An Inflationary Solution

This solution is obtained if we set H =constant. Thus Equations (1.2) and (1.3) give β = 6-3α

so that Λ = 6H^2. This can be integrated to give $H^2 = \Lambda/6$

$$\frac{\dot{R}}{R} = \sqrt{\frac{\pi}{6}} \qquad \text{where } R = \text{constant.}\exp\left(\sqrt{\frac{\pi}{6}}\,t\right) \ , \quad \rho_{\text{total}} = \rho_v$$

This is the familiar de-Sitter inflationary solution.

1.6 An Accelerating Universe

Now, we consider the case α = 0 i.e. $\Lambda = \beta\frac{\ddot{R}}{R}$

From Equations (1.2), (1.3) and (1.4), we find

$$\ddot{R} = \frac{-8\pi G}{3(n+1)}[1+\omega(n+1)]\rho R + \frac{\Lambda n}{3(n+1)}R$$

From Equations (1.9) to (1.15) shows that in the matter-dominated universe with G = constant, we have

$$R(t) = \left[\frac{(6-\beta)k_1 t}{3-\beta}\right]^{\frac{3-\beta}{6-\beta}}$$

$$\Lambda(t) = \left[\frac{3(3-\beta)\beta}{(6-\beta)^2\, t^2}\right]$$

$$\rho(t) = \frac{3(3-\beta)[3(n+1)-n\beta}{8\pi G(6-\beta)^2 t^2} \quad ,$$

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$$\rho_v(t) = \frac{3\beta}{8\pi G} \frac{[3-\beta]}{[6-\beta]^2 \, t^2}$$

$$q = \frac{3}{3-\beta}, \qquad \beta \neq 3$$

$$t_p = \frac{3-\beta}{6-\beta} H_p^{-1}, \qquad \beta \neq 3, \beta \neq 6$$

$$\Omega_p^m = \frac{[3(n+1)-n\beta]}{[(n+1)(3-\beta)]}$$

$$\Lambda_p = \frac{3\beta}{(3-\beta)} H_p^2 \ , \qquad \beta \neq 3$$

1.7 Conclusion

In this chapter, we have considered the decay law of the form $\Lambda = 3\alpha \frac{\dot{R}^2}{R^2} + \beta \frac{\ddot{R}}{R}$ $, where \ \alpha \ and \ \beta$ are dimensionless constants and found that cosmological constant Λ decreases as t^{-2} and the rate of particle creation is smaller than the steady state values in the framework of higher dimensional space time. Many higher dimensional models in the literature can be retrieved from this model with particular choice of α and β.

We have also shown that the universe must be accelerated if $\Lambda > 0$. This may be due to the fact that, if gravity is increasing, then the universe has to increase its expansion rate to escape the future collapse.

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